4.1 Undirected Graphs Glossary

Definitions.

A graph is a set of vertices and a collection of edges that each connect a pair of vertices.

A self-loop is an edge that connects a vertex to itself.

Two edges that connect the same pair of vertices are parallel.

When there is an edge connecting two vertices, we say that the vertices are adjacent to one another and that the edge is incident to both vertices.

The degree of a vertex is the number of edges incident to it.

A subgraph is a subset of a graph’s edges (and associated vertices) that constitutes a graph.

A path in a graph is a sequence of vertices connected by edges.

A simple path is one with no repeated vertices.

A cycle is a path with at least one edge whose first and last vertices are the same.

A simple cycle is a cycle with no repeated edges or vertices.

We say that one vertex is connected to another if there exists a path that contains both of them.

A graph is connected if there is a path from every vertex to every other vertex in the graph. A graph that is not connected consists of a set of connected components, which are maximal connected subgraphs.

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An acyclic graph is a graph with no cycles.

A tree is an acyclic connected graph.

A disjoint set of trees is called a forest.

A spanning tree of a connected graph is a subgraph that contains all of that graph’s vertices and is a single tree.

A spanning forest of a graph is the union of spanning trees of its connected components.

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Graph G with V vertices is a tree if and only if it satisfies any of the following five conditions:

■ G has V-1 edges and no cycles.

■ G has V-1 edges and is connected.

■ G is connected, but removing any edge disconnects it.

■ G is acyclic, but adding any edge creates a cycle.

■ Exactly one simple path connects each pair of vertices in G.

The density of a graph is the proportion of possible pairs of vertices that are connected by edges.

A bipartite graph is a graph whose vertices we can divide into two sets such that all edges connect a vertex in one set with a vertex in the other set.

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